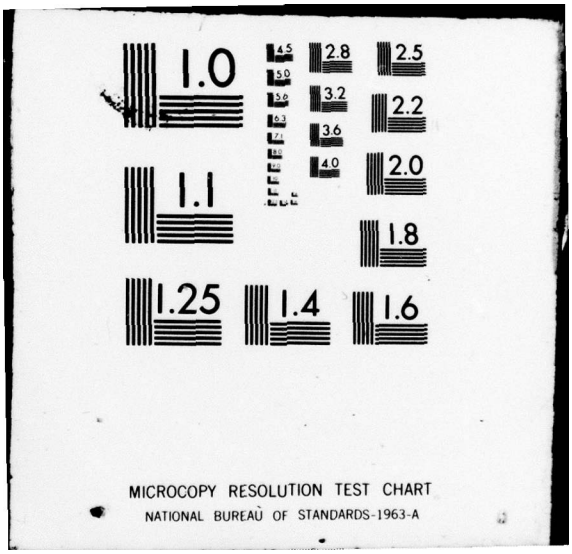
AD-A071 835 STANFORD UNIV CALIF F/G 14/4
RELIABILITY PREDICTION FOR MECHANICAL SYSTEMS.(U)
JUL 79 M V JOHNS DAAG29-76-G-0213
UNCLASSIFIED ARO-13859.3-MX NL

| OF |
AD
A071835

END
DATE FILMED
8-79
DDC


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0
4.5
5.0
5.6
6.3
7.1
8.0
9.0
MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

RELIABILITY PREDICTION FOR MECHANICAL SYSTEMS

FINAL REPORT

M. Vernon Johns, Jr.

July 11, 1979

U. S. Army Research Office

GRANT NUMBER: DAAG29-76-G-0213

STANFORD UNIVERSITY

APPROVED FOR PUBLIC RELEASE;
DISTRIBUTION UNLIMITED.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: FINAL REPORT
2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle): Reliability Prediction for Mechanical Systems.
5. TYPE OF REPORT & PERIOD COVERED: Final Report, 20 April 1976 to 19 April 1979
6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s): M. Vernon Johns, Jr.
8. CONTRACT OR GRANT NUMBER(s): DAAG29-76-G-0213

9. PERFORMING ORGANIZATION NAME AND ADDRESS: Stanford University, Stanford, California 94305
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: P-13859-M

11. CONTROLLING OFFICE NAME AND ADDRESS: U. S. Army Research Office, Post Office Box 12211, Research Triangle Park, NC 27709
12. REPORT DATE: July 11, 1979
13. NUMBER OF PAGES: 9

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office): 11 Jul 79
15. SECURITY CLASS. (of this report): Unclassified
15a. DECLASSIFICATION/DOWNGRADING SCHEDULE: NA

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

12 Apr.

18 ARO 19 13859.3-MX

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

NA

18. SUPPLEMENTARY NOTES

The findings in this report are not to be construed as an official Department of the Army position, unless so designated by other authorized documents.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Reliability, series system, confidence bound, Poisson distribution, robust estimation, Pitman estimator, efficiency.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Please see reverse side.

332 550 JOB

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

Two areas were investigated. (1) A method for producing exact confidence bounds for the reliability of highly reliable series systems. The methods are based on finding suitable orderings of the sample outcome space, and are valid when the probabilities of component failure are small enough for the failure data to be distributed as Poisson random variables. Optimality criteria are invoked to insure that the orderings used produce confidence bounds which are as tight as possible. Tables to facilitate applications were produced. (2) A new class of robust estimators of location were studied. These estimators (called P-estimators) are analogous to Pitman estimators in the same way that M-estimators are analogous to maximum likelihood estimators. Members of this class were evaluated by computer simulation and were found to perform with even higher efficiency than the "bisquare" M-estimator for both long and short tailed distributions.

SUMMARY OF PRINCIPAL RESEARCH ACCOMPLISHMENTS

The research effort supported by this grant has been concentrated in two main areas of investigation; (1) the determination of optimal exact confidence bounds for the reliability of systems consisting of components operating in series, and (2) the development of a new class of robust statistical estimation procedures analogous to Pitman estimators. The principal methods and results obtained are detailed briefly in the following two sections:

1. Reliability assessment for highly reliable series systems

Typically the reliability of a given series system must be estimated from failure data obtained from independent testing of the system components. Thus, the available data for a k component system may consist of x_i observed failures for component i resulting from n_i independent tests for that component ($i = 1,2,\ldots,k$). It is desired to produce a lower confidence bound on the reliability (probability of no component failures) for the system with prescribed confidence coefficient $1-\alpha$.

This fundamental problem has naturally received considerable attention in the literature (see, e.g., [8] for a summary) and many possible solutions have been proposed. All of these proposed solutions suffer from one or more of the following deficiencies; (i) the confidence coefficient is only approximately attained, (ii) the procedure is not computationally feasible, (iii) the procedure is not at least approximately optimal in some suitable sense, and (iv) the errors

inherent in approximations used are not known. The present investigation has largely surmounted these difficulties by (1) concentrating on the case of highly reliable systems (i.e., where the probabilities p_i of individual component failures are all small) so that the observed test failures can be modeled as Poisson random variables, (2) by developing confidence bound methods based on sample orderings and (3) by introducing concepts of minimum expected length for exact confidence bounds.

It has been known for some time (see, e.g., [2], [8]) that for certain problems, exact confidence bounds (i.e., bounds with true coverage probabilities $\geq 1-\alpha$) for a specified function of the relevant parameters may be constructed if the set of sample outcomes is endowed with a suitable ordering. In the present investigation conditions for the existence of such bounds were studied in a quite general framework, and the results were applied to the reliability problem for the case of Poisson observations. For the "highly reliable" case the numbers of component failures x_i, $i = 1,2,\ldots,k$, may be regarded as Poisson random variables with corresponding parameters $\lambda_i = n_i p_i$. Finding a lower confidence bound for system reliability turns out to be equivalent to finding an <u>upper</u> confidence bound $\theta^*(x_1,x_2,\ldots,x_k)$ for $\theta = a_1\lambda_1 + a_2\lambda_2 + \ldots + a_K\lambda_K$, where the a_i's are simple functions of the sample sizes n_i such that $a_1 + a_2 + \ldots + a_k = 1$. The lower $1-\alpha$ confidence bound for the system reliability is then given by $1 - \theta^*(x_1,x_2,\ldots,x_k)(\Sigma\ 1/n_i)$.

Aside from questions of computation, the problem reduces to that of finding an ordering of the set of possible sample points

$(x_1,x_2,\ldots,x_k)$ which will produce an upper confidence bound $\theta^*(x_1,\ldots,x_k)$ for θ having good properties. We first require that any ordering considered be consistent with the natural partial ordering which asserts that a sample point $(x_1,x_2,\ldots,x_k)$ is "as good as" (less than or equal to) another point $(y_1,y_2,\ldots,y_k)$ if and only if $x_i \leq y_i$ for $i = 1,2,\ldots k$. It is shown that any ordering which is consistent in this sense leads to a valid exact confidence bound for θ. In the course of this investigation several methods for generating orderings were studied. The final objective was to generate tables of values of the confidence bound for θ for $\alpha = .05$ and .10, for the case $k = 2$, for various values of $a_1 = 1 - a_2$. This, together with a simple procedure which reduces the cases where the number of components is three or more the $k = 2$ case, provides an easy accurate method for reliability assessment.

Various increasingly sophisticated (and computationally more and more demanding) methods were developed for generating suitable orderings and corresponding confidence bounds. The main optimality criterion introduced to determine the orderings was the minimization of the expected value of the upper bound θ^* under suitable prior distributions on the λ_i's. (Note: the bounds satisfying such criteria are still exact frequentistic confidence bounds, not Bayesian posterior bounds.) An optimal tree search method was developed to compute bounds satisfying the minimization criteria, but the core memory requirements became so enormous that only the first forty or so points in the optimal orderings could be determined. A sequential

procedure was then developed which starts at the origin and orders the sample points by looking ahead one stage (or two stages) and selecting the best candidate point subject to the partial ordering constraint. It was found that the two-stage "look ahead" procedure almost exactly reproduced the available portion of the optimal tree search ordering and could easily extend the orderings to the first one hundred points. It was further noted that the orderings were rather insensitive to the choice of prior distributions on the λ_i's so the final computations were performed for the limiting case of diffuse priors leading to uniform weights on the sample points. The resulting tables will appear in the revised version of a paper ([5]) originally submitted and provisionally accepted for publication in the Journal of the American Statistical Association.

2. Robust Estimation

Much attention has been devoted in recent years to the development of robust (or resistant) methods for estimating the parameters of models (see, e.g., [1] and [4] for surveys). Such methods are designed to give uniformly valid and efficient results when the underlying distribution of the observations can be either short-tailed like the normal distribution, long-tailed like the Cauchy or normal/uniform distributions, or intermediate in tail length like contaminated normal distributions. The most popular class of such methods are the so-called M-estimators introduced by Huber [3] which are based on generalizations of maximum likelihood

estimators. Since these estimators are typically required to be location and scale invariant, it seems plausible that the class of best invariant (i.e., Pitman) estimators might be generalized suitably to form robust estimators. Empirical and asymptotic investigation has shown that robust analogs of Pitman estimators may indeed be found and that their performance exceeds that of even the best M-estimators so far produced.

The general form of the Pitman estimator for location (e.g., the location of the center of a symmetric distribution) based on n iid observations $X = (X_1, X_2, \ldots, X_n)$ is

$$t(X) = \frac{\int_{-\infty}^{\infty} \theta \prod_{i=1}^{n} \gamma\left(\frac{X_i - \theta}{\sigma}\right) d\theta}{\int_{-\infty}^{\infty} \prod_{i=1}^{n} \gamma\left(\frac{X_i - \theta}{\sigma}\right) d\theta} .$$

If the X_i's have common probability density $\frac{1}{\sigma} \gamma\left(\frac{x - \theta}{\sigma}\right)$, then t(x) is the minimum variance location invariant (Pitman) estimator of θ. Robust location and scale invariant Pitman-like estimators of location (called P-estimators) are obtained by choosing a suitable function $\gamma(\cdot)$ (not necessarily a density) and replacing σ by an invariant estimator of scale S(x). As reported in [6], scale estimators S(x) which are themselves analogs of Pitman scale estimators lead to good robustness results. P-estimators have been exhibited which improve on the performance of Tukey's "bisquare" M-estimator by as much as four percentage points for both long and short-tailed distributions for n = 20.

Some additional results on robust estimators based on rank tests (R-estimators) have been obtained and are reported in [7].

REFERENCES

[1] Andrews, D.F., Bickel, P.J., Hampel, F.R., Huber, P.J., Rogers, W.H., and Tukey, J.W. (1972). Robust Estimates of Location: Survey and Advances. Princeton University Press.

[2] Buehler, R.J., "Confidence Limits for the Product of Two Binomial Parameters," Journal of the American Statistical Association, 52 (December 1957), 482-493.

[3] Huber, P.J. (1964). "Robust Estimation of a Location Parameter," Annals of Mathematical Statistics, 35, 73-101.

[4] Huber, P.J. (1977). Robust Statistical Procedures. SIAM Regional Conference Series in Applied Mathematics, No. 27.

[5] Johns, M.V. (1976). "Confidence Bounds for Highly Reliable Systems," Stanford Technical Report (to appear in J. Amer. Statist. Assoc.).

[6] Johns, M.V. (1979). "Robust Pitman-like Estimators," (to appear in the Proceedings of the ARO Workshop on Robust Statistics, April 1978, Academic Press.)

[7] Wesley, R.A. (1977). "Contributions to the Theory of Robust Regression," Stanford Technical Report, Stanford University.

[8] Winterbottom, A. (1974). "Lower Confidence Limits for Series System Reliability from Binomial Subsystem Data," Journal of the American Statistical Association, 69, 782-788.

PUBLICATIONS AND REPORTS

"Confidence bounds for highly reliable systems," (1976), M.V. Johns, Jr., Stanford Technical Report.

"Some aspects of reliability assessment based on sample orderings," M.V. Johns, Jr., Proceedings of the ARO Workshop on Reliability and Probabilistic Design, April 13-14, 1977, pp. 101-110.

"Robust Pitman-like estimators," M.V. Johns, Jr., Proceedings of the ARO Workshop on Robust Statistics, April 1978, Academic Press.

"Contributions to the Theory of Robust Regression," (1977), R.A. Wesley, Stanford Technical Report.

PARTICIPATING SCIENTIFIC PERSONNEL

Robert M. Bell

Barrett P. Eynon

David J. Pasta

Robert A. Wesley (Ph.D. granted 1978)